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ACCELERATION OF DEUTERONS IN ALVAREZ-TYPE PROTON LINAC

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I. Introduction

Ten years ago E. D. Courant pointed out a possibility of accelerating deuterons in the (old) AGS injector linac.[1] The ultimate goal was, of course, to accelerate deuterons in AGS for many interesting experiments. In principle, acceleration of deuterons in a proton linac is possible when the velocity of deuteron is everywhere one-half (in general $1/(2N)$, $N=1,2,3,...$) of the proton velocity. The length of each cell is then

$$L = 2\beta(\text{deuteron})\lambda$$

where $\lambda = 1.490$m with 201.25 MHz rf system. He demonstrated that, by simply changing the injection energy from 750 keV to 375 keV, one should be able to accelerate deuterons without any major change in linac. He concluded the discussion by making the following remark:

> "This mode of operation may be tried by seeing whether, with the preaccelerator at half normal voltage, the H_2^+ ions that always are produced to a certain extent in the ion source would be accelerated."

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It is not clear whether this experiment has ever been tried at BNL. Early in May, C. D. Curtis made a very preliminary investigation of H_2^+ acceleration at NAL but the result was negative. In this connection, it is interesting to note a statement made by N. D. West after Courant presented his paper:

> "N. D. West (Rutherford): I would like to point out that H_2^+ acceleration was tried with the Nimrod injector.[2] It seemed to us that the threshold rf field for acceleration was not very different from that of protons. We only spent about 10 minutes doing this and have not further investigated this."

Recently, deuterons were successfully accelerated (~1 mA) at Argonne in the ZGS injector[3] but details of the experiment are not available.

The purpose of this note is to find out whether one can accelerate deuterons (or H_2^+) in the injector linac at NAL without making any major modification of various systems. The result is rather discouraging. Out of 9 cavities, one can probably accelerate deuterons in the first three cavities (final energy 31 MeV). In cavities No. 4 to No. 7, the required rf voltage is such that one needs an order of magnitude more power than what is used now for proton acceleration. After cell No. 16 of cavity No. 6, the transit time factor becomes <u>negative</u> so that the synchronous phase must be switched from, say, -30° to +150° (+60° to +240° in the synchrotron convention). Particles are <u>decelerated</u> near the center of each gap and the acceleration is achieved in the area immediately before and after each drift tube.

II. Difference in Protons and Deuteron Accelerations

Each tank of an Alvarez-type linac can be regarded simply as a long cavity in which rf field of standing-wave type is established. Particles are electrically shielded when the electric field is in the decelerating direction. In terms of Fourier analysis of the axial electric field $E_z(z)$, the first harmonic component is responsible for proton acceleration and the second for deuteron acceleration. For a gap length (g) and a cell length (L), these components are proportional to

$$\sin(m\pi g/L) \Big/ (m\pi g/L) \qquad (1)$$

for a simple model and

$$J_o(m\pi g/L); \qquad J_o \equiv \text{Bessel function} \qquad (2)$$

for a somewhat better model of $E_z(z)$ in which the effect of a sharp boundary around the drift tube bore hole is taken into account.[4] Here m = 1 is for protons and m = 2 for deuterons. It is clear from these expressions that the rf accelerating field is less favorable to deuterons compared to protons. For a large value of (g/L), these quantities go to zero at (m = 2)

$$g/L = 1/2 \quad \text{or} \quad 0.383.$$

There is no acceleration in the cell, acceleration and deceleration cancelling each other exactly. If (g/L) is further increased, the sign of these components changes and, in order to maintain acceleration, one has to switch the phase of particles (relative to rf field) by 180°.

Another difference, which is important at low energies, is in the effect of the size of the bore hole. The accelerating component of rf field <u>on the axis</u> (r=0) is proportional to

$$1/I_o(K_m a); \qquad I_o \equiv \text{modified Bessel function} \qquad (3)$$

where a = radius of the bore hole, and

$$K_m \simeq 2m\pi/L. \qquad (L \ll \lambda)$$

The importance of this factor is evident in the following table.

	a (cm)	m=1	m=2
Original BNL, ANL	.635	.899	.673
LAMPF	.75	.864	.585
NAL, new BNL	1.0	.776	.414

(L = 6.037 cm, the first cell of linac)

Physically speaking, a larger bore size means less shielding of the undesirable rf field. Deuterons spend twice as much time compared to protons under the influence of this field, thereby getting less net acceleration. Acceleration of deuterons suffers from this effect at low energies and from large values of (g/L) at higher energies. For example, (g/L) of NAL linac goes up to 0.47 in the last cell of cavity No. 9.

III. Numerical Results for NAL Linac

The energy gain of the synchronous particle in each cell of a linac is given by

$$\Delta W = eVT \cos(\phi_s) \tag{4}$$

where V = voltage across the gap,

ϕ_s = synchronous phase (-32° for NAL),

and $T \equiv$ transit time factor = product of (1) or (2) and (3).

Since the velocity of deuteron is everywhere one-half of the proton velocity, the required energy gain ΔW for deuteron is approximately one-half (at low energy end) to one-third (at high energy end) of that for proton. Some of the relevant parameters are given in Table 1. The voltage across the gap is limited by the amount of rf power available and by the sparking. The synchronous phase may be reduced somewhat but this does not help much, $\cos(\phi_s)$ being already 85% of the maximum value (unity). Thus the crucial parameter for a successful acceleration of deuteron is the transit time factor T. Since expressions (1) and (2) are based on simple models for $E_z(z)$, they are not too reliable for NAL linac with rounded drift tube corners. Fortunately, all field values (E_z, E_r, H_ϕ) have been calculated numerically using the computer program MESSYMESH for each cell.[5] The accuracy of these field values should be better than a few percent in relative errors in view of the excellent result in the resonant frequency (~0.1%).

Transit time factor is given by

$$T_m = \int_{-L/2}^{L/2} E_z(z) \cos (2m\pi/L)dz \bigg/ \int_{-L/2}^{L/2} E_z(z)dz. \qquad (5)$$

The effect of the bore is already included since $E_z(z)$ is the field on the axis. The voltage (or field level) required for deuteron acceleration relative to that for proton acceleration is then ($E \equiv V/L$)

$$V_2/V_1 \equiv E_2/E_1 = (\Delta W_2 T_1) / (\Delta W_1 T_2) \qquad (6)$$

for the same value of the synchronous phase. Results are summarized in Table 2 and in Figs. 1-3.

Difficulties in cavities No. 4 - No. 8 are insurmountable. Even with an unlimited amount of power supply, one must switch the phase by 180° in cavity No. 6. The required field is already prohibitive near the end of cavity No. 3. However, one might be able to decrease ϕ_s in the last ten cells or so without losing the beam if the phase oscillation damping is normal. Cavity No. 1 is a challenge. Aside from the discontinuity at cell No. 18, the variation of E_o up to cell No. 20 may not be easy to realize. The dashed line in Fig. 1 is simply a suggestion for a trial when a special injection energy is being employed. This will be discussed in detail in the next section. Fortunately, ample supply of rf power is available in cavity No. 1 and the sparking limit is much higher than the dashed line.[6]

IV. Injection Energy

The dashed line in Fig. 1 is above the required average field for $\phi_s = -32°$ beyond cell No. 12. Near cell No. 18, it is slightly below for a few cells but this is not serious. The synchronous phase changes from -32° to -28° there and the resulting beam loss should be negligible. The major problem is how to keep the beam from the injection to cell No. 12. It should be remembered that, although the length involved is very short (0.82m), the phase oscillation wavelength is also short and particles make one complete phase oscillation. If the beam is injected at 375 keV, it will be lost almost entirely before reaching cell No. 12. A detailed numerical calculation shows that, for the injection energy between 370 keV and 380 keV, the entire beam is indeed lost before cell No. 12.

The synchronous energy of deuterons at the center of cell No. 12 is approximately 750 keV which is available from the present preaccelerator. In order for the beam to be captured in the stable area in cell No. 12 and be accelerated normally beyond, it is not necessary to have a net acceleration before this cell. The problem is then reduced to one of finding an area in the longitudinal phase space that will be transformed to the stable area in cell No. 12. Particle motions are rather complicated and numerical calculation seems to be the only reliable way to find the solution. MESSYMESH values for E_z at 340 points along the longitudinal axis from the linac entrance to the center of cell No. 12 have been used to integrate phase and energy for a given initial condition $0<W_i\leq 800$ keV and $-180°\leq\phi_i\leq 180°$. This calculation indicates that the collection of points (W_i,ϕ_i) that can eventually be captured in the stable area at the center of cell No. 12 makes a rather narrow, curved strip in phase space extending from slightly above 380 keV to almost 650 keV. The range in ϕ_i for a given value of W_i (phase acceptance) is generally very narrow ($\lesssim 15°$) but there are three groups of points that look promising. These groups are shown in Fig. 4 as (A), (B) and (C). Choice between (A) and (B) depends on the stability of the preaccelerator voltage. If the stability is better than ±1 kV at ~545 kV, (B) gives the largest phase acceptance (~150°). For a larger fluctuation, area (A) is better in overall acceptance, approximately 40° from 390 kV to 405 kV. Note that a buncher cannot increase the acceptance for (B) and may be of little use even for (A).

V. Conclusion

It seems very difficult to accelerate deuterons or H_2^+ in NAL linac beyond cavity No. 3 (31 MeV). Successful acceleration in cavity No. 1 is a result of a rather precarious balance of the rf field level, injection energy and the phase at the beginning of the cavity. It may be a painful task to achieve this balance in actual operation. Defocusing action of each gap for the betatron oscillation is twice as large for deuterons as for protons if the same synchronous phase is maintained.

However, beyond cell No. 12 of cavity No. 1, there should be no major difference in the quadrupole strength since momentum is practically the same. Between cell No. 1 and No. 12, the phase motion is "wild" so that the optimum quadrupole strength must be found from a detailed study of the phase oscillation. None of these points has been investigated so far.

Information supplied from time to time by C. D. Curtis, E. R. Gray, P. V. Livdahl, C. W. Owen, L. C. Teng and D. E. Young has been very valuable during the course of this study.

References

1. E. D. Courant, Minutes of the Conference on Linear Accelerators for High Energies (Brookhaven National Laboratory, August 20-24, 1962), pp. 289-293.

2. Nimrod injector

   injection energy = 600-650 keV, rf frequency = 115 MHz, final energy = 15 MeV, length = 13.5m, number of drift tubes = 49.

3. L. C. Teng, private communication.

4. L. Smith, Handbuch der Physik 44, pp. 341-389. The expression (1) is given in p. 368 and the expression (2) can be derived from $E_z(z)$ given in p. 349, Eq. (5.4).

5. I am grateful to D. E. Young and E. R. Gray for digging up this "priceless treasure."

6. C. W. Owen, private communication.

Table 1. Velocity, Kinetic Energy and Momentum of Protons and Deuterons for NAL Linac

Rest Mass	Protons 938.26 MeV			Deuterons 1875.6 MeV	
Cavity No.	β_{in}	W_{in} (MeV)	$(cp)_{in}$ (MeV)	W_{in} (MeV)	$(cp)_{in}$ (MeV)
1	.0400	.750	37.5	.375	37.5
2	.1478	10.42	140.2	5.14	139.0
3	.2747	37.54	268.0	17.95	260.1
4	.3570	66.18	358.6	30.61	340.2
5	.4141	92.55	426.9	41.55	397.0
6	.4569	116.54	482.0	50.95	440.1
7	.4913	138.98	529.2	59.29	475.3
8	.5204	160.53	571.8	66.92	505.5
9	.5452	181.01	610.3	73.84	531.5
Final	.5665	200.31	645.0	80.09	553.9

Synchronous phase is -32° for all cavities.

Table 2. Average Field Required for Deuteron Acceleration

T_1 = transit time factor for protons on the axis

T_2 = transit time factor for deuterons on the axis

E_2/E_1 = average field required for deuterons relative to that for protons

Cavity	Cell	T_1	T_2	E_2/E_1 $\phi_s = -32°$	E_2/E_1 $\phi_s = 0°$
1A*)	1	.644	.194	1.656	1.405
	2	.656	.206	1.594	1.352
	3	.668	.218	1.532	1.299
	4	.680	.230	1.476	1.252
	5	.691	.241	1.427	1.211
	6	.701	.253	1.384	1.174
	7	.710	.264	1.345	1.141
	8	.719	.275	1.303	1.105
	9	.728	.287	1.266	1.074
	10	.736	.297	1.231	1.044
	11	.743	.307	1.207	1.024
	12	.750	.317	1.179	1.000
	13	.756	.326	1.154	.979
	15	.767	.342	1.119	.949
	17	.777	.358	1.084	.919
1B*)	18	.741	.303	1.212	1.028
	19	.747	.311	1.193	1.012
	27	.780	.359	1.078	.914
	35	.798	.386	1.025	.869
	45	.807	.395	1.006	.853
	56	.805	.380	1.036	.879

Cavity	Cell	T_1	T_2	E_2/E_1 $\phi_s = -32°$	$\phi_s = 0°$
2	1	.860	.538	.776	.658
	15	.860	.530	.784	.665
	30	.849	.492	.818	.693
	45	.830	.436	.888	.753
	60	.808	.371	1.000	.848
3	1	.823	.413	.911	.773
	10	.805	.361	1.003	.851
	20	.783	.302	1.142	.969
	35	.750	.215	1.491	1.265
4	1	.747	.208	1.537	1.304
	15	.717	.136	2.199	1.865
	29	.687	.0668	4.153	3.522
5	1	.732	.169	1.739	1.475
	10	.714	.126	2.237	1.897
	24	.686	.0622	4.194	3.557
6	1	.684	.0581	4.504	3.820
	10	.668	.0223	11.13	9.437
	16	.657	-.0002		
	22	.646	-.0239	-9.808	-8.318**)
7	1	.643	-.0292	-8.009	-6.792
	10	.628	-.0589	-3.773	-3.200
	21	.611	-.0910	-2.323	-.1970
8	1	.608	-.0960	-2.199	-.1865
	10	.595	-.119	-1.686	-1.429
	20	.580	-.147	-1.301	-1.103

Cavity	Cell	T_1	T_2	E_2/E_1 $\phi_s = -32°$	$\phi_s = 0°$
9	1	.577	-.152	-1.266	-1.073
	10	.566	-.169	-1.089	- .924
	19	.554	-.188	- .939	.797

*) In cavity No. 1, the cell geometry changes at cell No. 18.

**) $\phi_s = 148°$ or $180°$ for $E_2/E_1 < 0$.

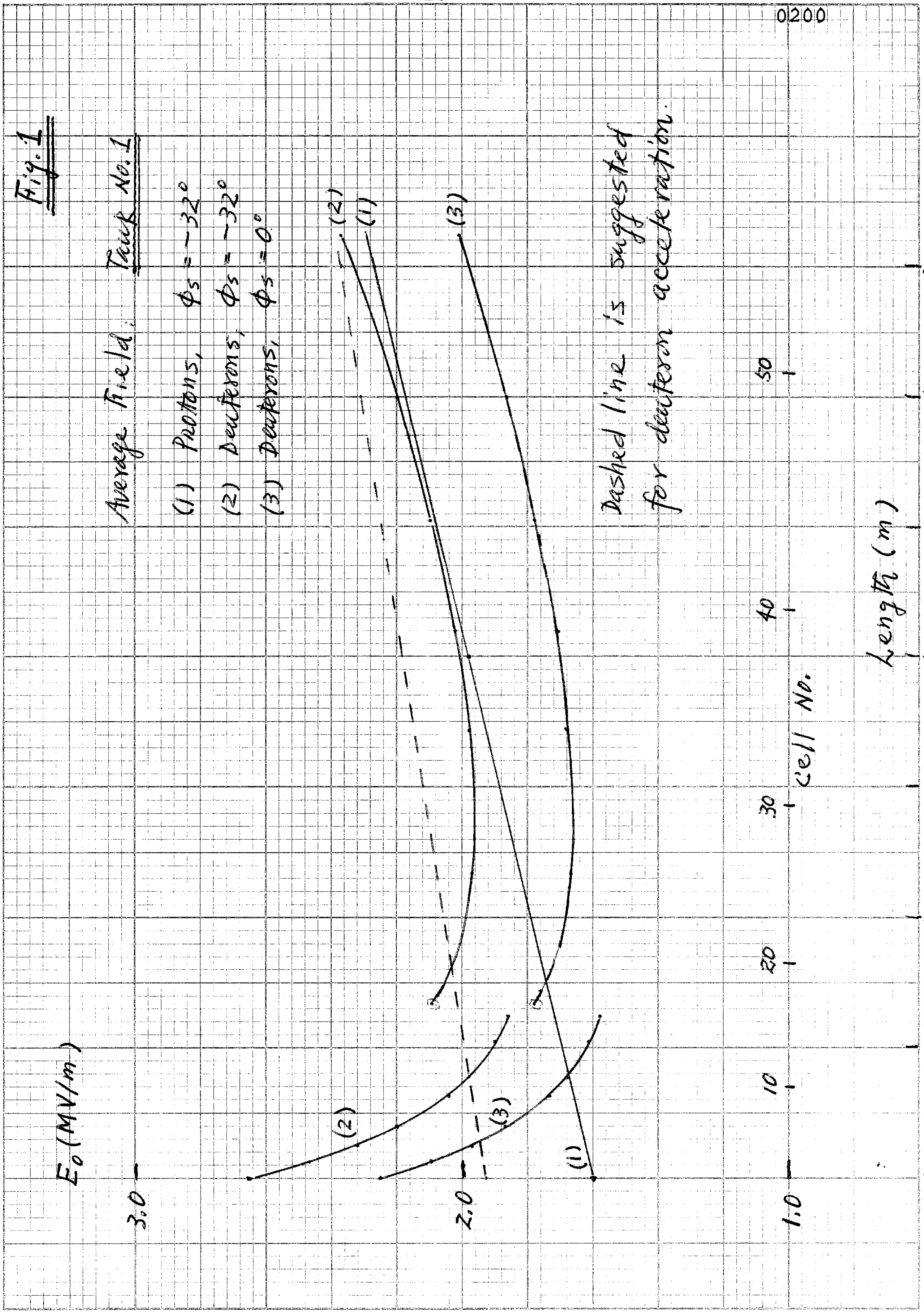
Fig. 1
Average Field; Tank No. 1
(1) Protons, $\Phi_s = -32^\circ$
(2) Deuterons, $\Phi_s = -32^\circ$
(3) Deuterons, $\Phi_s = 0^\circ$
Dashed line is suggested for deuteron acceleration.
E_0 (MV/m)
3.0
2.0
1.0
Cell No.
10
20
30
40
50
Length (m)
0
1
2
3
4
5
6
7
(1)
(2)
(3)

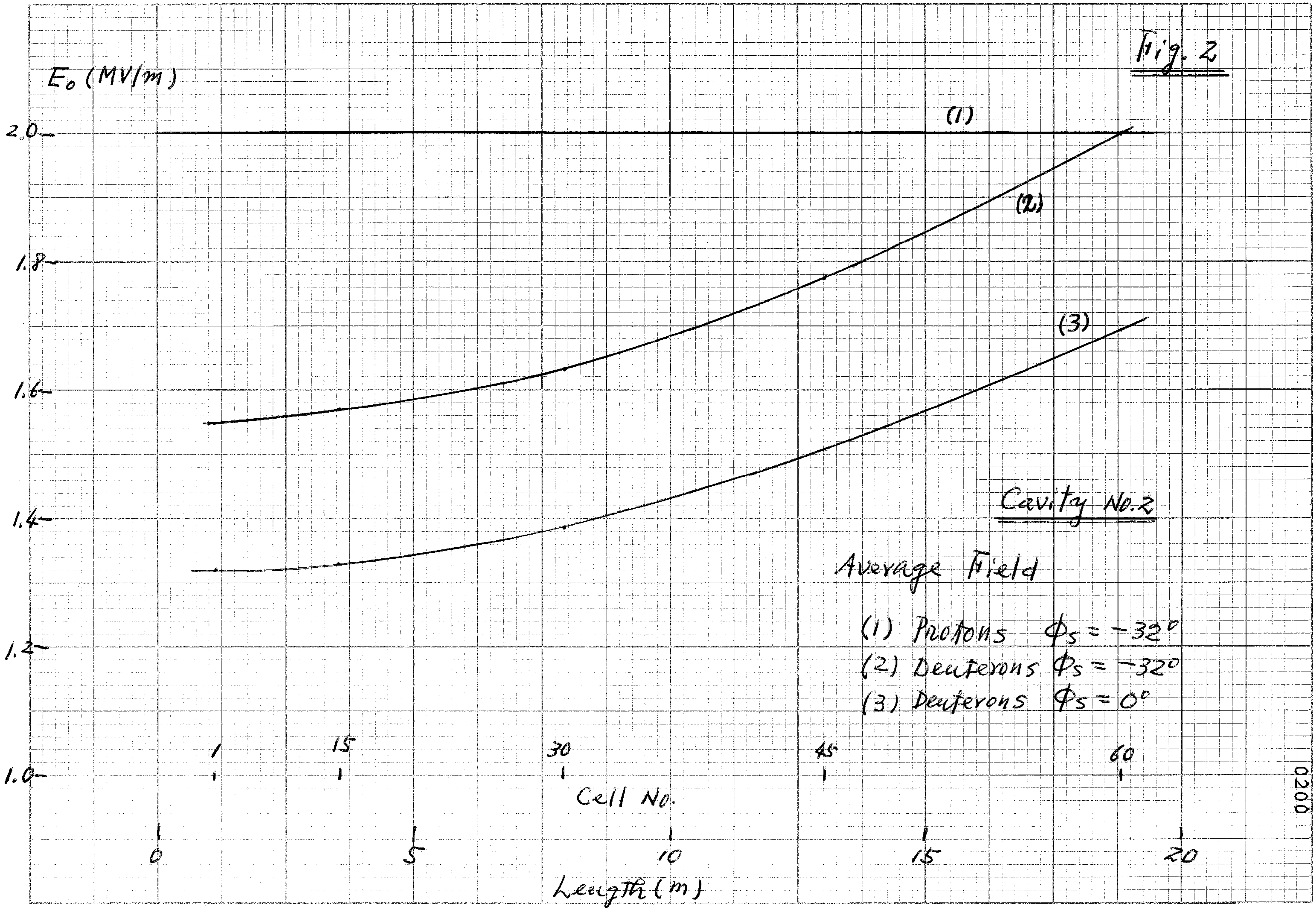
Fig. 2
E_0 (MV/m)
2.0
1.8
1.6
1.4
1.2
1.0
(1)
(2)
(3)
Cavity No. 2
Average Field
(1) Protons $\phi_s = -32°$
(2) Deuterons $\phi_s = -32°$
(3) Deuterons $\phi_s = 0°$
1
15
30
45
60
Cell No.
0
5
10
15
20
Length (m)

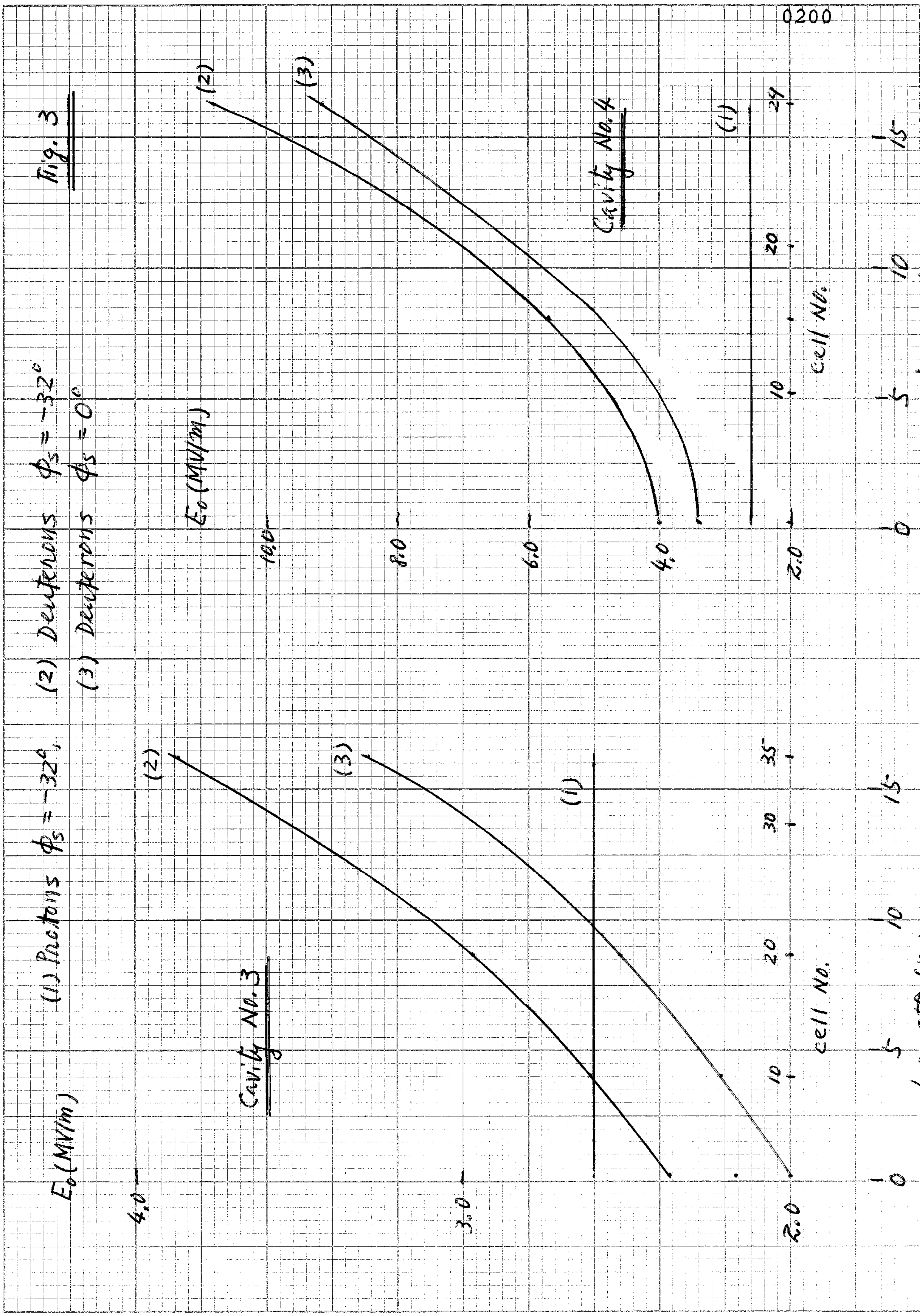
Fig. 3
(1) Protons $\phi_s = -32^\circ$,
(2) Deuterons $\phi_s = -32^\circ$
(3) Deuterons $\phi_s = 0^\circ$
Cavity No. 3
Cavity No. 4
E_0 (MV/m)
cell No.
Length (m)
2.0
3.0
4.0
6.0
8.0
10.0
0
5
10
15
20
29
30
35
(1)
(2)
(3)

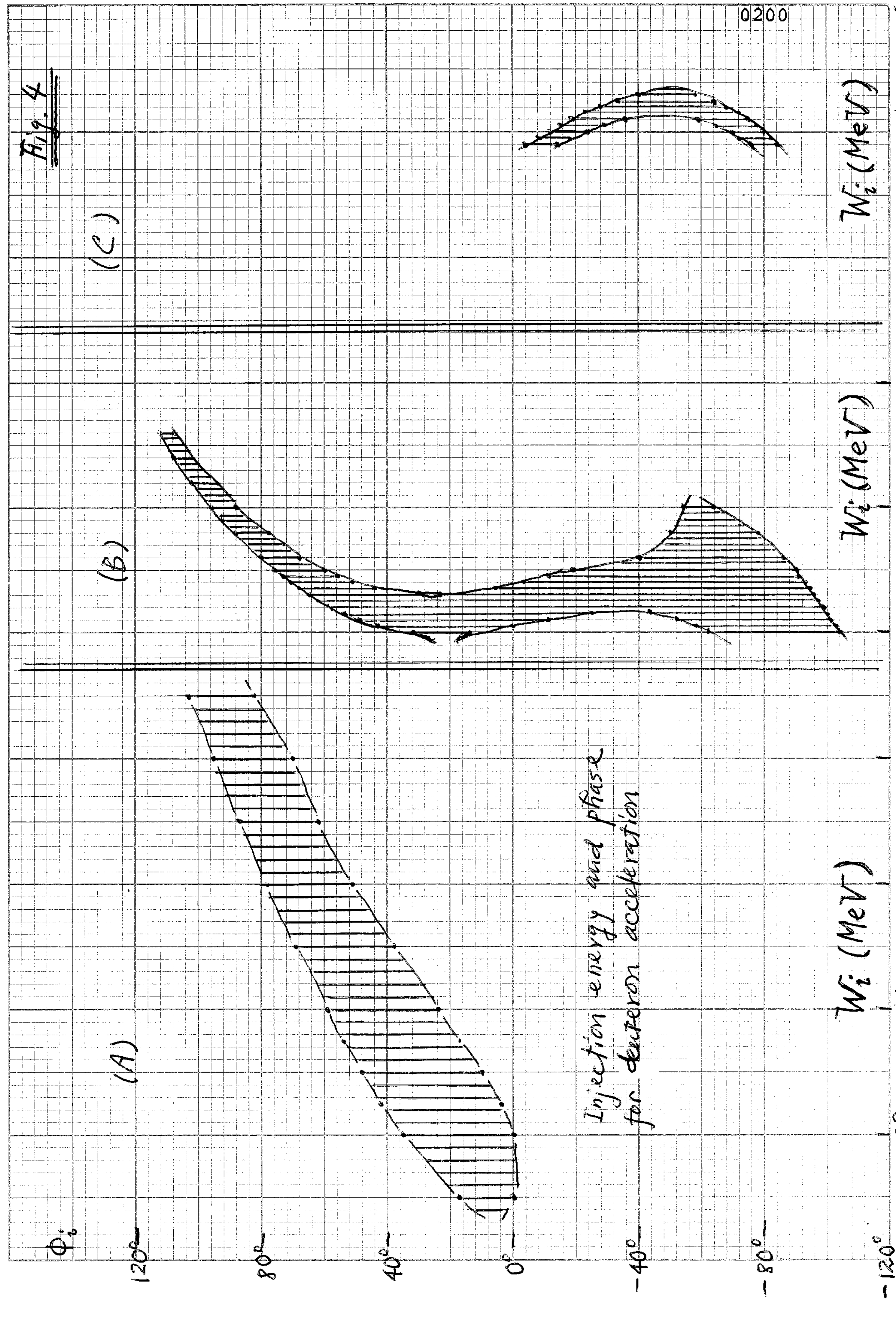
Fig. 4
(A)
(B)
(C)
Injection energy and phase for deuteron acceleration
Φ_i
120°
80°
40°
0°
-40°
-80°
-120°
W_i (MeV)
W_i (MeV)
W_i (MeV)
.39
.40
.41
.42
.54
.55
.56
.63
.64
.65